

Rather than trying to keep idealization to a workable minimum, however, Weir seems happy to go in the other direction. To summarize in his own words,

So long as there exist concrete tokens of the theorems [and proofs?] ... of the applied mathematics which one needs in the idealization, then it is permissible for a neo-formalist to use the theorems of the idealizing theory. ... [This] reveals that the emphasis on the finite nature of proof which took hold in logic after Gödel's work in the 1930s is a harmful prejudice which should be abandoned. There should be no size limit on the abstract structures which can be invoked in idealizations of our finite corpus of mathematical utterances. (p. 10)

Which — to return to an earlier issue we raised — gives us an idea of how the Gödelian gap between truth and provability (as idealized) is supposed to be closed: just appeal to an infinitary idealization which is not recursively enumerable so the incompleteness theorem does not apply.

Yet *this* will strike many readers as a rather astonishing turn for the formalist to take. We started with what seemed to be a robust emphasis on the honest toil of concrete proofs in an arithmetic system *S* (whatever it is) as making for arithmetical correctness. And now we are invited to see another source of arithmetical correctness, one still arising from concrete proofs, but now proofs in a (wildly!) idealizing applied mathematics modelling the producing of *S*-proofs. Which might look like a very considerable revision in the notion of arithmetical truth. Or perhaps — some will think — this just reflects what is in the end the rather slippery open-endedness and unclarity in the notion of proof that is supposed to make for arithmetical truth. Either way, I suspect many readers initially tempted by other aspects of Weir's neo-formalism will in the end find his pivotal chapter seven on idealization a real sticking point.

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*In Defence of Objective Bayesianism*, by Jon Williamson. Oxford: Oxford University Press, 2010. Pp. xiii + 198. H/b £46, \$85.

## 1. Introduction

*In Defence of Objective Bayesianism* by Jon Williamson is an admirably ambitious book. The book is quite technically sophisticated and ably explains and accommodates both the successes and failures of more 'classical' versions

of the Bayesian approach to inference while developing a novel and interesting version of that approach. I am not persuaded that this version is entirely successful, but Williamson's arguments are subtle, interesting, and worthy of serious consideration.

## 2. Background and summary

By way of setting the context, here is an all-too-brief summary of the state of play with regard to modern Bayesianism before the publication of Williamson's book. Nearly all Bayesians are united in their acceptance of Probabilism, the view that credences are rationally constrained by the axioms of the probability calculus (for example, the constraint that the rational credence to assign to a disjunction of mutually exclusive propositions is the sum of the credences assigned to each disjunct). Moreover, nearly all Bayesians are united in their acceptance of the Bayesian Rule of Conditionalization, which says that, when you acquire new evidence  $E$ , your new credence in each proposition  $q$  should be your old credence in  $q$  conditional on  $E$ ;  $p_{new}(q) = p_{old}(q|E)$ , where conditional credences are calculated using Bayes's Theorem. Bayesians differ with regard to which additional constraints beyond Probabilism and Conditionalization they accept; Subjective Bayesians usually claim that there are no further rational constraints on credence, whereas Objective Bayesians insist on further constraints imposed by symmetry considerations, or 'Reflection' on the subject's future rational credences, or the known objective chances of future events, etc. Subjective Bayesianism has struck many as 'too subjective'; after all, if all I know is that an urn contains some red balls and some black balls, it would seem crazy to have a credence of (say) .999 that the next ball randomly drawn from the urn will be black. But neither have Objective Bayesians had widespread success in articulating or defending their additional constraints; for instance, the most notorious way of spelling out the rational constraint imposed by symmetry considerations, the Principle of Indifference, has been shown to be inconsistent by Bertrand-style paradoxes such as van Fraassen's 'cube factory' case (see van Fraassen, *Laws and Symmetry*, Oxford: Clarendon Press, 1988, chap. 12).

Perhaps the most notable feature of Williamson's view is that, though he accepts Probabilism (in the form of a norm he calls **Probability**), he *rejects* one of the centrepiece of classical Bayesianism — namely, the Bayesian Rule of Conditionalization. Let us not quibble over whether such a view still deserves to be called 'Bayesian'; I will just refer to Williamson's view as 'Williamsonianism'. In the place of Conditionalization, Williamsonianism endorses the two norms **Calibration** and **Equivocation**. According to these norms, the agent's credences should satisfy constraints imposed by her evidence (**Calibration**) and should be otherwise equivocal among the basic possibilities that she can express (**Equivocation**). This rejection of Conditionalization in favour of **Calibration** and **Equivocation** is quite

significant: whereas most versions of Bayesianism endorse Conditionalization as an *update rule* that takes the evidence *and the agent's old credences* as inputs and generates as outputs the new credences that the agent should have, Williamsonianism entails that when an agent acquires new evidence, she should simply *throw out* her old credences and apply the three norms, afresh, to her new evidential situation. So, it is not just that Williamsonianism rejects Conditionalization as the *particular* rationally required update rule; Williamsonianism is more radical in rejecting the existence of *any update rule at all* that takes any of the agent's prior credences as inputs. Of course, just like classical Bayesianism, Williamsonianism allows for a subject's credence in a proposition to increase (or decrease) when she acquires new evidence for (or against) that proposition; after all, more evidence for *q* will mandate (via **Calibration**) a higher credence in *q*. But, according to Williamsonianism (unlike classical Bayesianism), that is not because the new evidence mandates an *increase* from *q*'s old credence *as such*; rather, it is just because the new evidence mandates (via **Calibration**) a new credence which, as it happens, is higher than the old one.

### 3. Insensitivity to evidence

One odd feature of Williamsonianism is that, precisely because it severs the rational tie between pre-evidential and post-evidential credences, there are situations where Williamsonianism recommends no change in credence where a change would seem to be warranted.

To take an example that Williamson returns to at several points in the book, suppose that a doctor is trying to determine a prognosis for a patient who has been treated for breast cancer, and suppose that all the doctor knows is that breast cancer recurs in between 10% and 40% of cases. Let *R* be the proposition that the cancer will recur. **Probability** constrains the doctor to have credences in *R* and  $\neg R$  that sum to 1, and **Calibration** constrains the doctor to have a credence in *R* that is somewhere in between .1 and .4. **Equivocation** constrains the doctor to have credences in *R* and  $\neg R$  that are as close as possible to the maximally equivocal assignment of .5 to each proposition; thus, Williamsonianism requires the doctor to have a credence in *R* of .4 and a credence in  $\neg R$  of .6.

Even supposing that this is all correct, the oddness arises if we suppose that the doctor later learns that, in fact, breast cancer recurs in 40% of cases (suppose that all the doctor remembered earlier was that the recurrence rate was somewhere in between 10% and 40%, and then later she looked it up in the relevant books and saw that the rate is in fact 40%). Williamsonianism entails that the doctor's credences should remain precisely the same; after all, given the new evidence, **Calibration** constrains the doctor to have a credence in *R* of .4, and **Probability** then constrains the doctor to have a credence in  $\neg R$  of .6 (and **Equivocation** no longer plays any role). But

this strikes me as counterintuitive; I would have thought that, when the doctor looks up the recurrence rate for breast cancer and finds that it is at the *very highest end* of the range that she considered possible, that would be grounds to *increase* her credence in R. Or, to dramatize the point differently, if my doctor were to tell me in the morning that the chance of my cancer recurring was between 10% and 40%, and then called me back in the afternoon to tell me that he had determined that the chance was in fact 40%, I would regard the afternoon call to contain some rather bad news.

A related problem arises when we consider violations of (something in the neighbourhood of) van Fraassen's Reflection Principle that are sanctioned by Williamsonianism (van Fraassen, 'Belief and the Will', *The Journal of Philosophy*, 81, 1984, pp. 235–56; 'Belief and the Problem of Ulysses and the Sirens', *Philosophical Studies*, 77, 1995, pp. 7–37). Suppose again that all the doctor knows about the recurrence rate of breast cancer is that it is between 10% and 40%, but suppose that the doctor knows that the exact rate is written in his notebook (call this rate  $x$ ), which is sitting across the room, and which he plans to look inside of presently. The doctor knows that, once he looks in the notebook, **Calibration** will constrain him to assign a credence to R that matches the value of  $x$ . So, the doctor's current credence in R is .4, even though he knows that in a few seconds, his credence in R will be .4 at the highest, and may well be significantly lower. Again, I find this situation very odd; knowing that in a few seconds, I may well (rationally) decrease my credence in R, and that I definitely will not (rationally) increase my credence in R seems to be a reason to decrease my credence in R *right now*, contra Williamsonianism. Williamson is aware of a possible tension with Reflection and in section 4.4 rejects Reflection principles that make use of conditional credences in favour of Reflection principles that appeal only to the norms of Williamsonianism, since arguments for Reflection that appeal to conditional credences assume the truth of Conditionalization. But as far as I can tell, there is no appeal to Conditionalization in my objection above; all that my objection requires is that, if the agent were to learn that the value of  $x$  is .3 (say), then the uniquely rational credence for her to have in R would be .3, which Williamson accepts. So, it seems to me that Williamsonianism violates even Williamson's own formulation of Reflection.

#### 4. Williamson's critique of conditionalization

Williamson deploys many arguments in favor of Williamsonianism and against Conditionalization, and argues that Williamsonianism is to be preferred where the two procedures conflict. Some of these arguments, I think, are not particularly persuasive. For example, in section 4.2.2, Williamson argues that if an agent acquires new evidence (call this E) that the physical chance of  $q$  is .8, Williamsonianism (via **Calibration**) correctly entails that the agent's credence in  $q$  should be .8, whereas Conditionalization agrees

with this verdict only if the agent's pre-evidential conditional credence  $p(q|E)$  was .8; Williamson claims further that the classical Bayesian is under no obligation to set his  $p(q|E)$  to .8. But all this shows is that anyone—Williamsonian or not—should accept some principle akin to Lewis's Principal Principle, which forces an agent's credences to conform to the known physical chances (except in cases where the agent has 'inadmissible' information). And nearly all Bayesians do accept such a principle. So this argument strikes me as targeted against a straw-man version of the classical Bayesian view.

A second relatively unpersuasive criticism of Conditionalization comes in section 4.2.4, where Williamson considers the case of a standard fair die being thrown. The agent's prior credences are subject to the constraint (call this constraint C) that the expected score for the die is 3.5 (i.e.  $(1+2+3+4+5+6)/6$ ). Then, the agent learns that the outcome is even, and Williamson points out that Conditionalization entails that the agent's posterior credences must violate C, since Conditionalization entails that the agent will have posterior credences of  $1/3$  each in the propositions that the die landed 2, 4, and 6, and hence entails that the expected score for the die is 4 (i.e.  $(2+4+6)/3$ ). But in this case, it seems completely obvious to me (even leaving any Conditionalization sympathies I may have aside) that C is a constraint only on the agent's prior credences, and not on her posterior credences; the fact that the die landed on an even outcome *should* increase the agent's expectation of the score, since the outcome space is finite, and the even outcomes can be placed in one-to-one correspondence with the odd outcomes such that each even outcome is higher than its corresponding odd outcome.

However, chapter 4 also contains some much more interesting considerations in favour of Williamsonianism and against Conditionalization. One common worry about Conditionalization is that it does not permit an agent's credence in any proposition to change from 0 or 1; once your credence in a proposition has taken on an extremal value, no possible evidence could ever rationally permit you to change it. Similarly, while Conditionalization does permit an agent's non-extremal *unconditional* credences in individual propositions to change over time (as she collects new evidence), it does *not* permit an agent's *conditional* credences to change, ever. And this really is a liability of Conditionalization; as Williamson points out, learning that data were produced by a non-independent process rather than an independent one might well give you reason to change lots of your conditional credences, such as your conditional credence that the 100<sup>th</sup> coin will land heads, given that the first 99 landed heads.

Williamson entertains other examples as well of pieces of evidence that are 'non-simple' with respect to one's background evidence in this way—i.e. by justifying a change in one of the agent's conditional credences. In each case, Williamson argues that, since Conditionalization forbids changes in

conditional credences, Conditionalization is unable to deliver the correct verdicts; by contrast, since Williamsonianism denies that new credences are rationally beholden to old ones, it permits any new conditional credences at all that are compatible with its three norms.

However, it was a bit hard (for me, anyway) to have very strong intuitions about the cases that Williamson discusses where the relevant evidence is non-simple with respect to the agent's background evidence. Here is one of the cases that Williamson considers:

Suppose A is 'Peterson is a Swede,' B is 'Peterson is a Norwegian,' C is 'Peterson is a Scandinavian,' and E is '80% of all Scandinavians are Swedes.' Initially, the agent sets  $p(A) = .2$ ,  $p(B) = .8$ ,  $p(C) = 1$ ,  $p(E) = .2$ , and  $p(A \wedge E) = p(B \wedge E) = .1$ . ... Updating by [Williamsonianism] on learning E, the agent believes that Peterson is a Swede to degree .8, which seems quite right. On the other hand, updating by conditionalizing on E leads to a degree of belief of .5 that Peterson is a Swede, which is quite wrong. (p. 80, notation slightly modified for simplicity)

On the one hand, Williamson is right that there is some intuitive force to the thought that, after learning E, the agent's credence in A should be .8; after all, E says that 80% of Scandinavians are Swedes. On the other hand, the agent thought that  $A \wedge E$  and  $B \wedge E$  were equally likely, so there is also some intuitive force to the idea that once she learns that E is true, she should split her credence evenly between A and B, and hence should believe that Peterson is a Swede to degree .5; after all, this case is different from one in which the agent's  $p(A \wedge E) = .16$  and her  $p(B \wedge E) = .04$ , and it is at least somewhat plausible that this difference should matter. So I do not think that the intuitive verdict about this case is as clear as Williamson claims, and given the paramount importance of this sort of case in adjudicating between Conditionalization and Williamsonianism, I would have liked to have seen a deeper discussion of the relevant intuitions.

## 5. Conclusion

Of course, there is a great deal more to discuss in *In Defence of Objective Bayesianism*, which for reasons of space I cannot pursue here. Williamson's discussion in chapter 9 of the Maximum Entropy Principle as a precisification of **Equivocation** and its putative advantages over the classical Principle of Indifference is particularly interesting, as is his application of his view to an impressive range of problems in chapters 5–8. There are other objections that I have to Williamson's particular formulation of his view, but I think that the most significant thesis of *In Defence of Objective Bayesianism* is that it is a worthwhile research program to pursue the Bayesian approach to inference, freed of the Conditionalization norm, and fleshed out instead in the 'consider-the-evidence-afresh'-style that Williamsonianism develops. And it clearly is. Each style has its advantages over the other and the matter is far from settled, but that is precisely what makes the research programme worth

pursuing. And Williamson's book has taken us one important step forward in this pursuit.

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*Meaning in Life and Why it Matters*, by Susan Wolf, with an introduction by Stephen Macedo, comments by John Koethe, Robert M. Adams, Nomy Arpaly, and Jonathan Haidt, and responses by Susan Wolf. Princeton, NJ: Princeton University Press, 2010. Pp. xvii + 143. H/b \$25.95.

This book consists of two chapters originally presented as the Tanner Lectures in Princeton in November 2007, together with comments given on that occasion and Wolf's replies. It seems a little odd that it is Princeton, not the Tanner Foundation, which is bringing out this book, which, while it expresses thanks to the Tanner Foundation, actually fails to say that these were Tanner Lectures.

The first lecture is on 'Meaning in Life', and argues that lives can be assessed not just in terms of their happiness and their dutifulness, but also in terms of the meaningfulness of the pursuits undertaken. A happy and upright life might still be squandered on trivial pursuits. Wolf takes meaning to arise in a life from love of objects worthy of love—through positive engagement with such objects. The sort of meaning a life may have is not like that a word or sentence has, which can be given in other words, and is 'the meaning'. Other words for this meaning which a life may or may not have are richness and depth, while verbal meanings need be neither rich nor deep. Wolf is surely right that we do assess human lives this way, as well as in terms of their happiness and moral worth, but I think she is wrong in saying (p. 7) that academic philosophers 'do not talk much about meaningfulness in life'. When referring to Bernard Williams, she says he is one of the few contemporary philosophers who have seen the relevance of meaning (p. 55). But there are many such philosophers. As the entry on the meaning of life in the Stanford Encyclopedia of Philosophy by Thaddeus Metz shows there have been plenty of books and anthologies on this topic, and philosophers from Pascal and Schopenhauer to Karl Britton, Thomas Nagel, Owen Hanfling, Harry Frankfurt, John Cottingham, and Kurt Baier have all addressed the topic. (Baier has provided fairly extended discussion in 'The Meaning of Life' (inaugural lecture, Canberra, 1957) and *Problems of Life and Death; a Humanist Perspective*, Amherst: Prometheus Books, 1997.) Wolf alludes to none of these. It is one thing to ignore predecessors, especially in two brief lectures, but another and a worse thing to deny their existence.